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NZQA

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MANA TOHU MĀTAURANGA O AOTEAROA

For Supervisor's use only

Level 3 CAS Calculus, 2009

90835 Demonstrate an understanding of patterns and relationships when solving problems

Credits: Five
2.00 pm Thursday 26 November 2009

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Make sure you have a copy of the Formulae and Tables Booklet L3–CALCF.

You should answer ALL the questions in this booklet.

Show ALL working for ALL questions.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–14 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

For Assessor's use only	**Achievement Criteria**				
Achievement		**Achievement with Merit**		**Achievement with Excellence**	
Demonstrate an understanding of patterns and relationships when solving problems.		Demonstrate a deeper understanding of patterns and relationships when solving problems.		Demonstrate a comprehensive understanding of patterns and relationships when solving problems.	
Overall Level of Performance					

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You are advised to spend 50 minutes answering the questions in this booklet.

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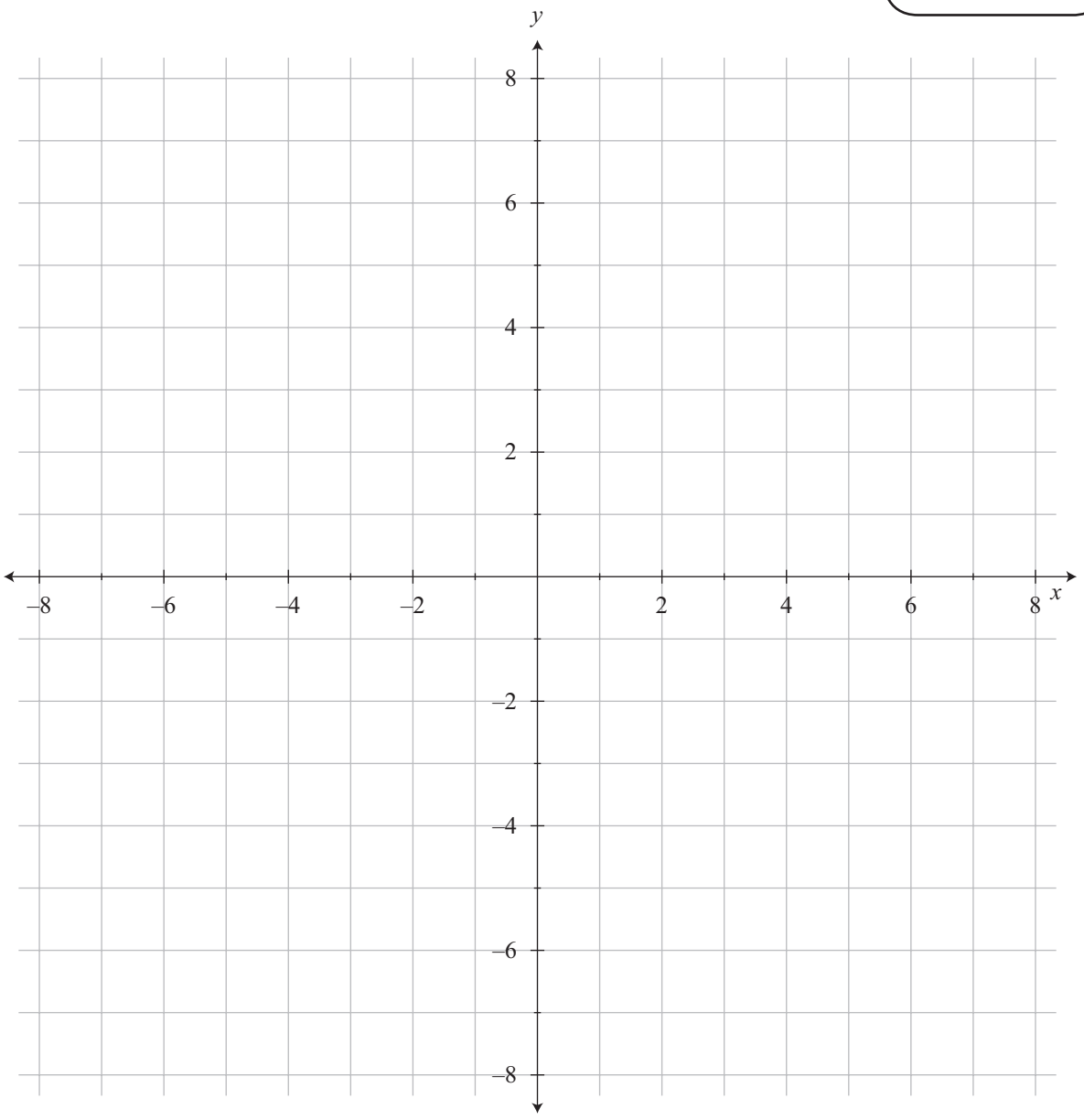
QUESTION ONE

(a) Sketch the graph of the curve defined by $y = 4t$ and $x = 2t^2 - 2$.

Label any intercepts and asymptotes.

If you need to redraw this graph, use the grid on page 13.



(b) Write the equation of the conic section shown below.

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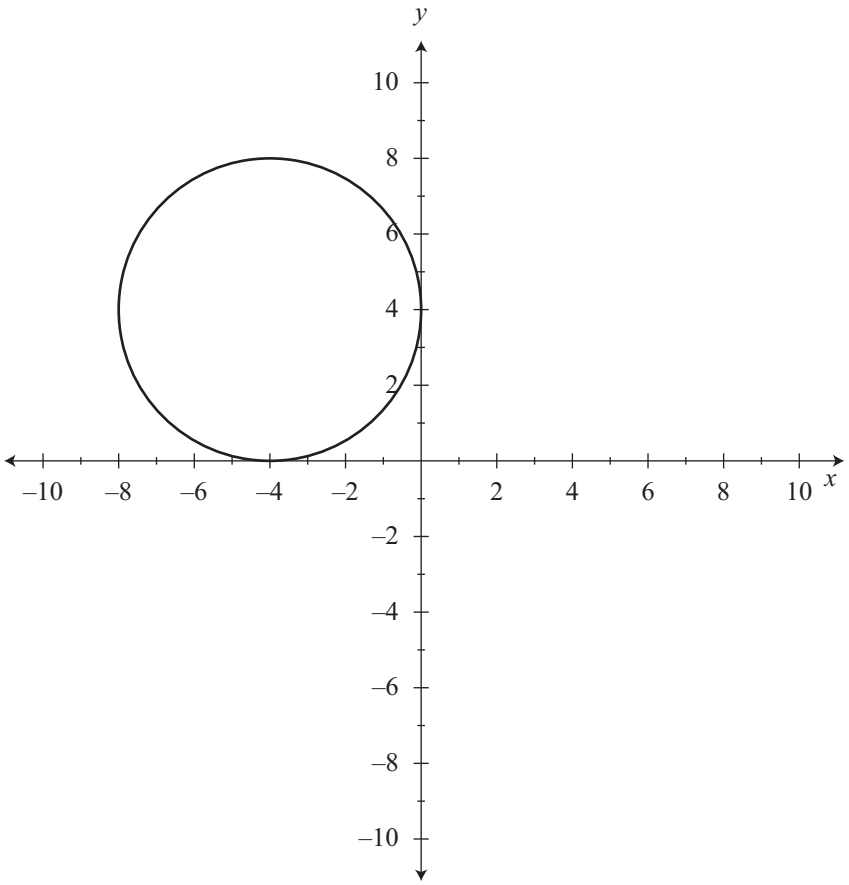

(c) Write the equation of the conic section shown below.

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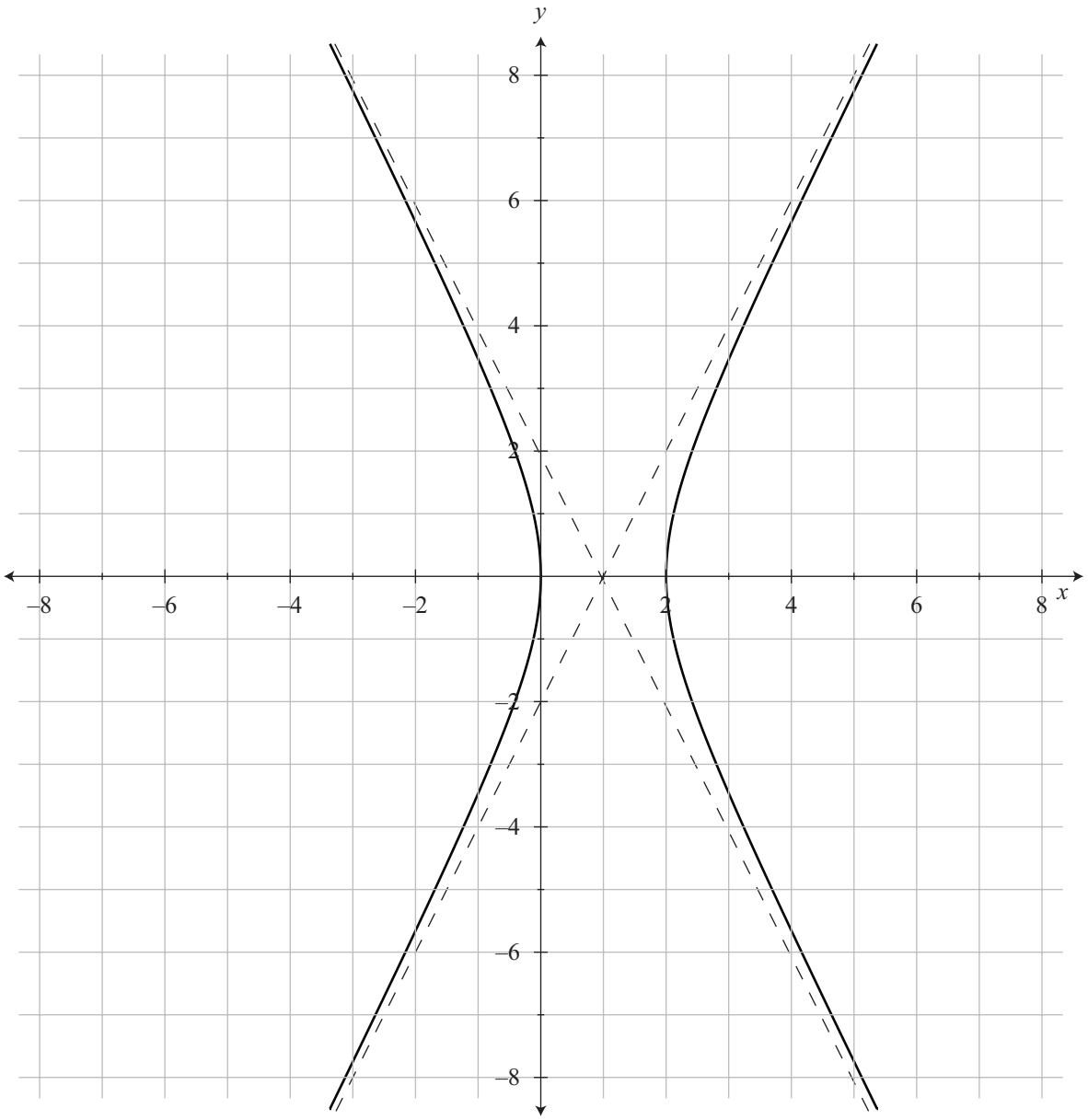

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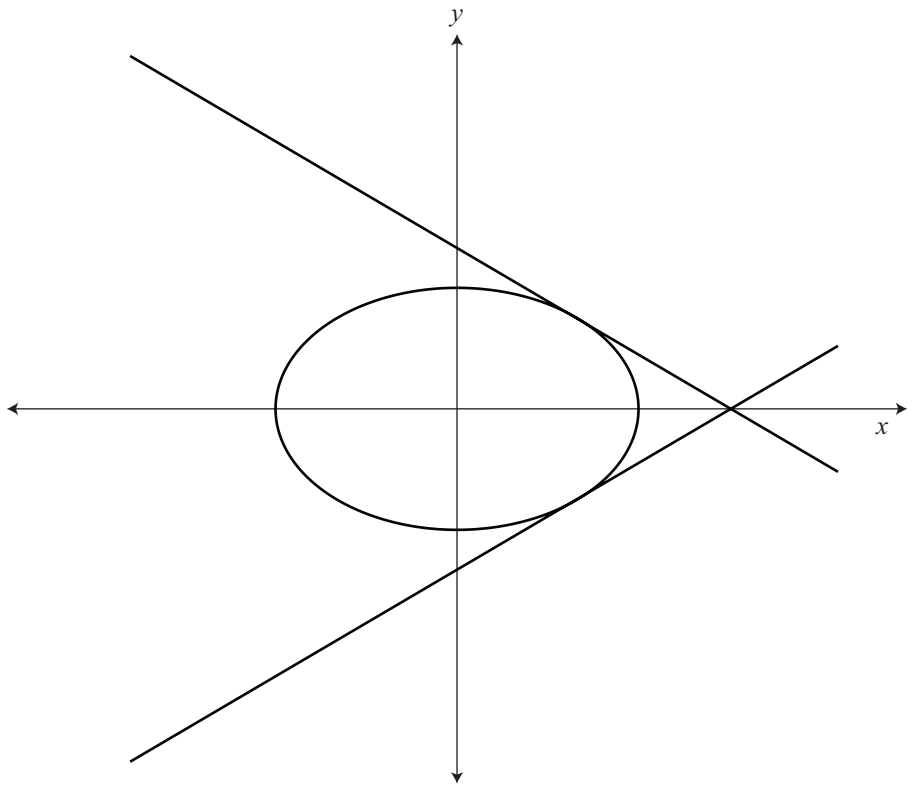
(d) An ellipse has the equation $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

Tangents are drawn at $x = 2$.



Find the **exact** gradients of the tangents.

(e) Find the x-intercept of the tangents in part (d) on the opposite page.

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(f) An ellipse has the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where $a > b > 0$.

A tangent to the ellipse is drawn at (m,n) in the upper half of the ellipse.
The tangent cuts the x-axis at $(c,0)$.

Show, by finding the gradient of the tangent or otherwise, that

$$\frac{a^2}{b^2} = \frac{mc - m^2}{n^2}$$

Hint: You may find it helpful to sketch a graph.

QUESTION TWO

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(a) The motion of Jan's bicycle pedal can be modelled by a sine function.

The height of the pedal above the ground can be modelled by the equation

$$h = 17\sin\frac{8\pi t}{3} + 28$$

where h = the height in cm of the pedal above the ground
and t = time in seconds since Jan started cycling.

How long will it take for 100 revolutions of the pedals?

(b) Jan's bicycle has a red flashing light on the back. The intensity of the light varies from 0% to 100%. It can be modelled by the equation

$$I = 50 - 50\cos\frac{4\pi t}{3}$$

where I = intensity (%)
and t = time in seconds since the light was turned on.

Will the light be increasing or decreasing in intensity after 2 seconds?

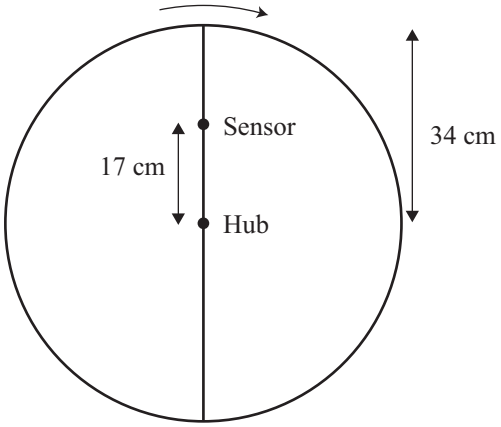
Show your working.

Assessor's use only

(c) Jan's bicycle has a sensor attached to the back wheel to measure the number of times the wheel rotates each minute.

When Jan is cycling on level road, the sensor records 195 wheel revolutions per minute.

The sensor is attached at a distance of 17 cm from the hub, and the wheel has a radius of 34 cm.



Write a trigonometric equation that will model the motion of the sensor, where h is the height in cm above the ground, and t is the time in seconds after 195 revolutions per minute is reached.

(d) The path of a comet can be modelled by a parabola with equation $y^2 = 2x$.

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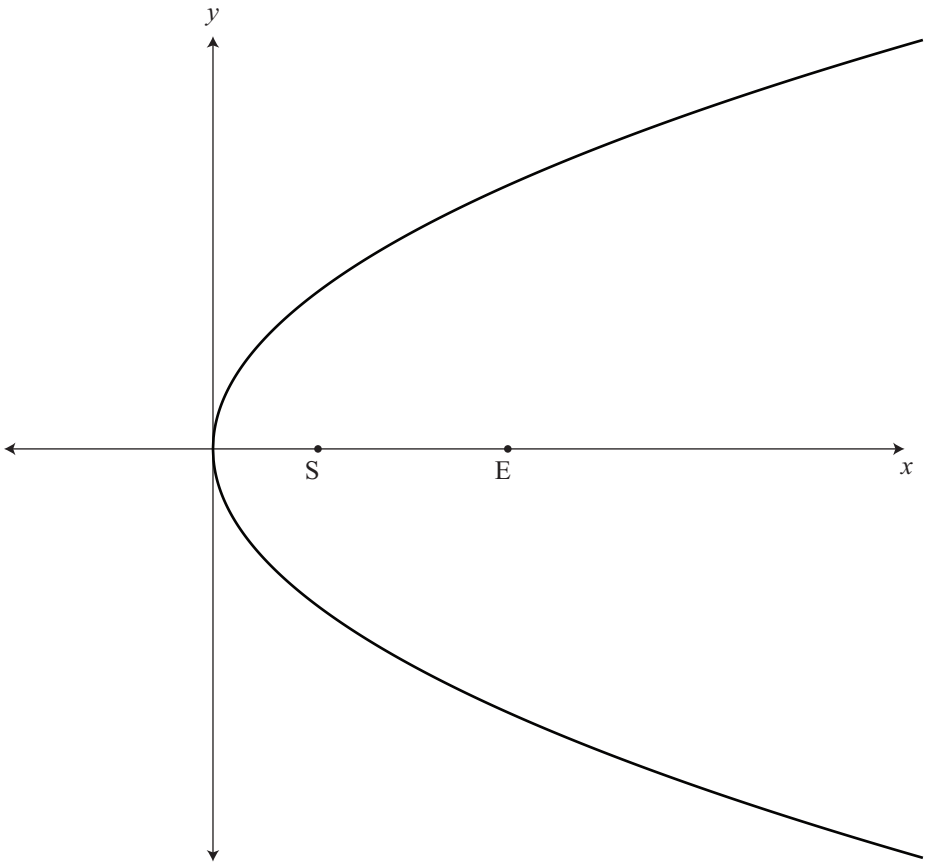
The Sun, at S, is at the focus of the parabola, and the Earth, at E, lies along the axis of symmetry of the parabola at a distance of 1 Astronomical Unit from the Sun.



What is the closest the comet gets to the Earth?

Notes: The movement of the Sun and the Earth can be ignored.
An Astronomical Unit (AU) is the average distance from the Earth to the Sun.

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If you need to redraw the graph from page 2, draw it on the grid below and carefully number the question. Make sure it is clear which graph from the question you want marked.

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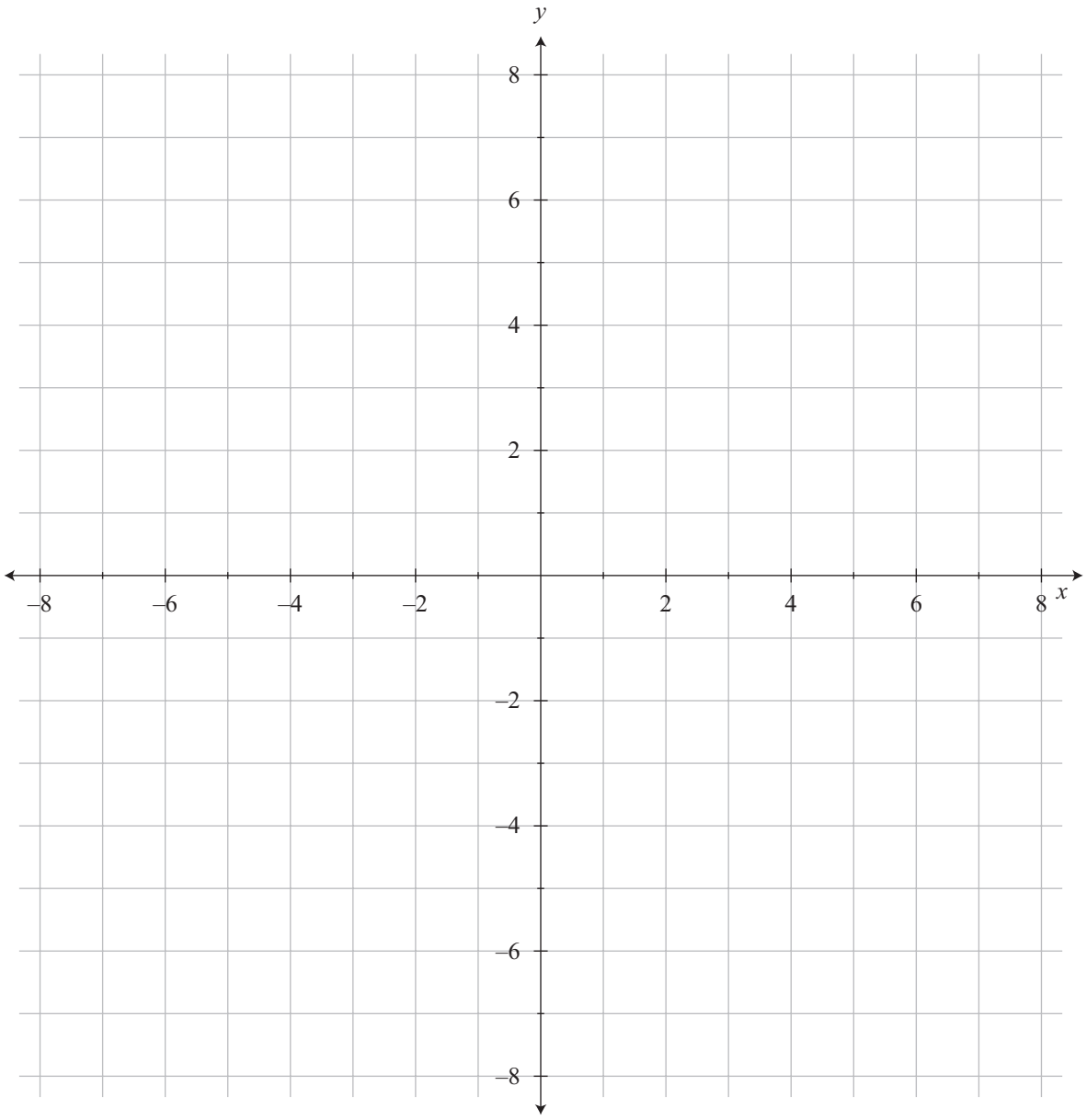
Question _____



Extra paper for continuation of answers if required.
Clearly number the question.

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Question number	

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